Ore Reserve Evaluation and Open Pit Planning

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SYNOPSIS

The computer methods used by the French Commissariat à l'Energie Atomique, which are applicable to any type of ore deposits, are:

(i) Ore reserve estimation. The data are either the core analyses or gamma loggings. To determine the spatial distribution of the ore, three different methods of interpolation are used. Two of these are based on the least-squares theory and the third one is an application of the regionalized variable theory. The interpretations of the orebody by the geologists are introduced in the computer program as often as possible.

(ii) Optimal open pit design. From the spatial distribution of ore, the three-dimensional outlines of the optimal pit are determined by a geometrical algorithm which allows different slopes varying with the azimuth.

Finally, the optimization of annual successive plans are discussed.

INTRODUCTION

The method used to arrive at the best open pit design and the program of exploitation from exploration results are described.

The system of working is as follows:

(i) From the grade logs of boreholes, we test for the positions of different mining limits depending upon minimum mining height, minimum waste width and cut-off grade. For uranium, we are able to do all these calculations directly from gamma-logs. The method used in this case is described in Appendix A.

(ii) From the position of the mining limits found in the boreholes, we build a model of the payable orebody.

(iii) From this model, we look for the best open pit design and the best program of exploration, considering slope stability and economy.

All calculations are done by computer but after each stage, we check the results and discuss them with the specialists of the different technical departments.

METHOD OF DETERMINING THE MINING LIMITS

Grades of boreholes are established from samples of any length. If these lengths are unequal, we first calculate grades of imaginary samples of the same length, assuming that the grade of a real sample is uniform over its full length, that the grade of non-sampled zones is null and that, for a null cut-off grade, total metal contents calculated from the imaginary samples and from the real samples are the same (Fig. 1).

In fact, this method produces a slight smoothing effect on the grades. We limit this loss of contrast by using for the length of the imaginary samples a value near to the average length of the real samples.

From the consecutive samples, we compute mining width using an algorithm which, depending on a given cut-off grade, G_c, a given minimum width and a minimum waste width, selects the optimum mineable width in the borehole. This is achieved by calculating for each of the n samples (grade G_i) the difference $G_i - G_c$ and testing for the maximum of the sum $G_i - G_c$ depending on the given mining method. If this maximum is negative there is no possibility of mineable widths in the borehole. Usually we assume that minimum mining width and minimum widths are the same because they will be mined with the same machine.

Rapid evaluations of the ore reserves are obtained by multiplying the values obtained for mineable width for each borehole by an influence zone to allow us to observe the variations of ore and metal quantities and of average grade, depending upon cut-off grade, mining width and waste width. These calculations assist the economists in choosing the values of cut-off grade, minimum mining width and minimum waste width for use in building the different models of the economic orebody.

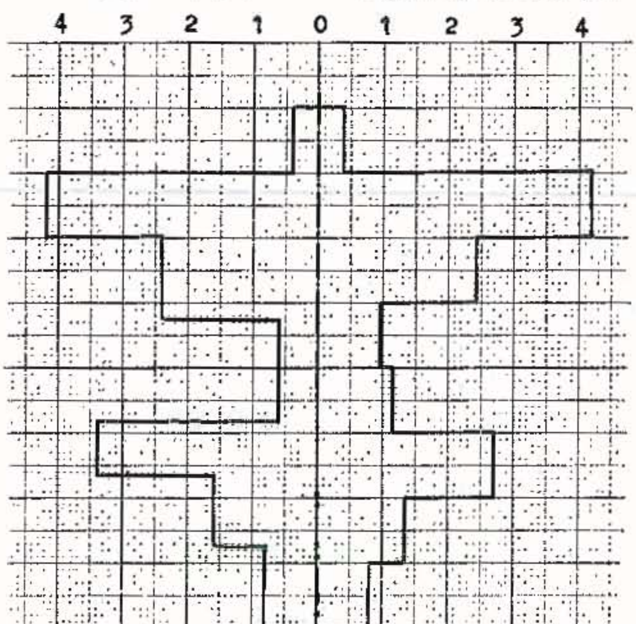

Fig. 1. Real and imaginary grades.

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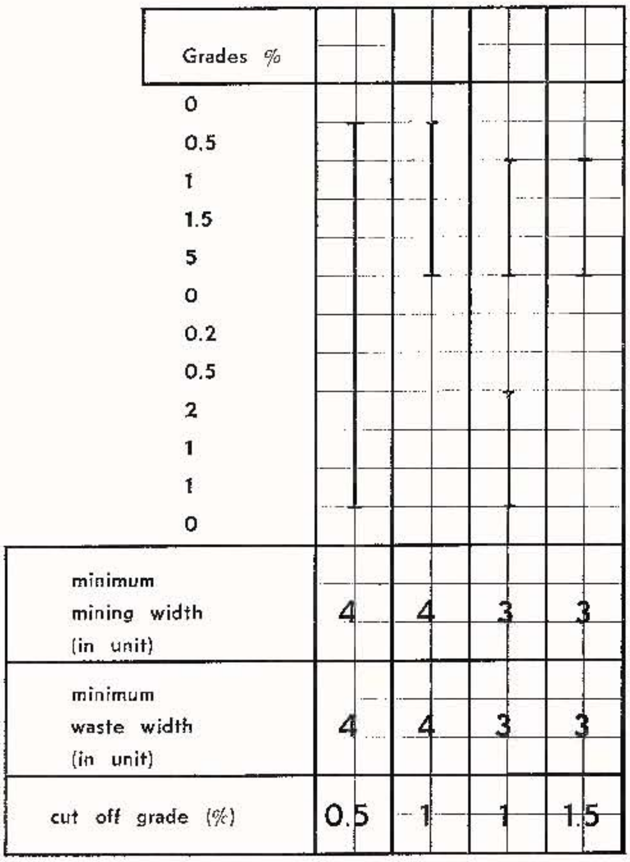

Fig. 2. Mining limits for different mining conditions.

CONSTRUCTION OF AN ECONOMIC OREBODY MODEL

INTERPOLATION

Here the methods of interpolation for an orebody formed by a single bed (or a vein), mined at one mineable width, are explained. When there are several mineable widths, we assume that the orebody comprises a series of beds. The optimum exploitation method is then more complex to explain but not really different from the optimum exploitation method for a one-bedded orebody.

The interpolated values are referred to a three-dimensional co-ordinate grid, the x and y axes defining a horizontal plane.

So, for a one-bedded orebody with one economic horizon, we know the mineable width in each borehole and also where the mineralized horizon outcrops. In place of the outcrops and the unpayable boreholes we put in imaginary figures with mining width equal to the minimum one and grade null. This avoids changing the geological continuity of the bed.

By interpolation, we calculate for the orebody:

(i) the depth of the economic horizon below surface,

(ii) the width of the mineable horizon, and

(iii) the grade thickness (grade thickness = average grade × height).

The values are plotted on a rectangular grid for each variable function.

Interpolation by the use of a trend surface

This method was used only at the beginning of our work to test the optimal open pit program rather than to model an orebody.

This method produces a smoothing in a three-dimensional space by fitting an n-order trend surface. The testing for best fit of the surface was by the method of least squares. We used up to 10th-order surfaces, that is, 66 coefficients.

This method of calculation has many well-known drawbacks:

(i) smoothing is overdone,

(ii) with irregular grid data, numerous spurious anomalies with high order surfaces.

Interpolation by the use of vertical-axis paraboloids

In this method we locally adjust the three-variable function by a vertical-axis paraboloid.

As we need at least six points to define a vertical-axis paraboloid, we fix the maximum number (N) of points that we wish to occur in a rectangular area, the dimensions of which depend on the density of data and also on the anisotropy of the studied variable. If N is equal to six, we find the paraboloid passing through these six points, or else if N is more than six we look for a paraboloid best fitting the points, best fit being defined by the method of least squares.

Inside the rectangular area, the center of which is the point at which we want to know the interpolated value of the variable, we test for the real values (boreholes, outcrops) where this variable is known.

We then count the number of known points inside the rectangular area.

(i) If this number is less than six, calculation is impossible in the center of this area.

(ii) If this number is between six and N, we test whether these points are not all in one half of the area. If this is so, we would in effect have an extrapolation, so we stop the calculation at this point. In all other cases we calculate the vertical axis paraboloid.

(iii) If this number is greater than N, we use only the N points closest to the center and we calculate as above.

We find this method very good for any kind of variable, even for one presenting rapid variations. However, any geologist will doubt its validity because of the arbitrary choice of the dimensions of the rectangular areas and of the number N.

Interpolation using regionalized variable theory.

This method also known as universal kriging was originated by G. Matheron (1970). First, the spatial interrelationships of the variable are studied. This involves the half-variogram, which can be transformed to the autocorrelation function. So we get a very important feature of the variable: the influence zone of each unit of information.

When the half-variogram is calculated, we use universal kriging. This means that we choose, for a point on the grid, the estimator whose variance is minimum. This estimator is optimal.

As for the paraboloid method, we never calculate the estimator at a point where it will give an extrapolation of the real values.

If an intersection on the grid is right on a point where the experimental value of the variable is known, universal kriging gives this exact experimental value. This property is valuable. The other methods, with the exception of a six-point paraboloid, do not have this property.

At each point, universal kriging also gives, for several types of half-variograms, the variance of the interpolated values indicating the points where calculation is doubtful.

For more details on the methods, the publications of G. Matheron quoted in the bibliography should be consulted.

MODEL CHECKING

Once the interpolations of depth, mining width and grade thickness have been done, we check that the economic criteria are met at every point, that is, whether the mining width is greater than minimum width and metal content is greater than cut-off grade.

From the interpolated data, using an incremential digital plotter, we draw maps showing:

(i) depth below surface of the economic horizon,

(ii) mining width, and

(iii) average grade.

These maps are then given to the geologists so that they can check the model and eventually correct it.

The interpolation methods we describe are all unbiased. The last two give excellent results. Each time they were used the geologists had to correct only the border values.

All open pit design implies a good knowledge of the topographic surface and, on occasion, of other surfaces related to depth, for example, depth of the altered zone where the mining machinery will be different from that used in coherent rocks. These surfaces are calculated by means of the interpolation methods described.

PROBLEMS SET BY OPEN PIT MINING

The techniques described above permit definition of the areas of mineralization and are of particular interest for open pit mining projects.

The French uranium mining industry includes a notable proportion of open cast mines; planning methods have, therefore, had to be conceived and developed for these mines.

The following questions must be answered in connection with the exploitation of deposits in shallow depth by open pit mining:

(i) Which part of the deposit can be exploited by the open pit method (the remainder being suitable for underground mining or rejected)?

(ii) In what chronological order should the deposit be exploited?

(iii) What is the economic value of the project?

These questions are related by the desire for optimization which the economic system forces industrial enterprises to seek.

Generally, a project may consist of two parts:

(i) *the analysis of the economic system*, and

(ii) within this framework, the search for *the optimum pit design* and the *optimum exploitation strategy*.

ANALYSIS OF THE ECONOMIC CLIMATE

Any project presupposes a sufficient knowledge of the economic climate. Obviously for a mining project the major factor is the *level of prices*; however, some other aspects of this environment must not be forgotten, such as local outlets for the product, means for financing, fiscal regime, terms of supply, possibilities of recruiting staff, etc.

The economic climate can best be considered through a criterion of choice or *general criterion of optimization*. This criterion of choice is a method permitting several possible solutions to the problem of exploitation to be classified in order of merit. The power of human decision is an example of a criterion of choice which offers in particular the ability to take qualitative elements into account. The *discounted cashflow* is the most often used objective criterion. To clarify the rest of the paper, the general criterion of optimization will be called hereafter the 'discounted cashflow'.

The calculation of the value of the variable to be optimized for a project (value of the project) is generally rather tedious. This is due to the non-linearity of some of the functions, especially those related to time (actualization) and to volume (economies of scale). Therefore, the use of a computer is absolutely necessary to obtain a rapid answer. Several projects carried out by the Commissariat a L'energie Atomique have necessitated the development of *programs for detailed financial analysis*.

RESEARCH INTO OPTIMUM PIT DIMENSIONS

Aims

Very often the exploiter can detail the plans for stripping and ore production only if he already knows the definitive outline of the open pit mine. This is the case when he imposes on himself technical constraints of the following kind:

(i) excavate from the start over the full area, deepening during the working life without any lateral extension;

(ii) maintain a constant rate of stripping.

Consequently, the successive stages of the investigation may consist of:

(i) defining an open pit contour according to a certain optimization lagorithm;

(ii) inferring a plan for exploitation, and

(iii) calculating the discounted cashflow of the total project.

Except in exceptional cases, the hypothesis introduced in the algorithm to determine the contours does not maximize the discounted cashflow at the first attempt. Therefore, an iterative procedure will have to be used, with changes in the hypothesis.

General scheme of the optimization algorithms

Introduce an *evaluation function*, $V(x, y, z)$, which is defined at all points within the mineral deposit of volume D. The algorithm used for optimization must permit the determination of that portion of D, say C, for which:

(i) the shape of C corresponds to a *technically* exploitable open pit, that is, the banks are stable at least during the period of exploitation; obviously the slopes of these banks must be less than those of the slopes defined by research into rock mechanics or soil mechanics,

(ii) the integral $V = \int_C v(x,y,z)dxdydz$ is *maximum*.

Usually, we know only the trend in v in terms of other parameters, such as the change in the grade, the selling price of the metal, etc. Hence, we define v as a simple function (generally linear) of these characteristics (grade, prices, etc.) and one or several arbitrary parameters.

For example, in the very simple case where the materials do not present differential mechanical characteristics and the ore has a practically uniform grade, a convenient function v may be:

$$v(x,y,z) \left\{ \begin{array}{ll} = a & \text{for any element of ore} \\ = -1 & \text{for any element of waste,} \end{array} \right.$$

where a is the parameter to be optimized.

The solutions given through the application of the algorithm for a limited series of values of these *optimization parameters* are then examined to infer a plan of working, and evaluated by calculation of the discounted cashflow.

However the most critical point of this research is the determination of the open pit C which maximizes the integral V. A great variety of algorithms of optimization have been used to give suitable answers to the problem.

Manual method

This classical graphic method in which plans and cross-sections with calculation of volumes by planimetry are used, illustrates the optimization parameters and supplies a comprehensive example of the optimization algorithm.

The contours of the mineralization at a given *cut-off grade* are drawn on plans (first optimization parameter); generally, the main bodies of mineralization supply a minimum open pit which is then enlarged by the examination of any mineralized extensions. These extensions are workable if they can be removed with a *stripping ratio* less than a fixed value (second optimization parameter). This process takes into account the layout of the haulage road, which means it must be used when the influence of the road is dominant (small open pits). The biggest problem is, in the case of important pits, the length and the cost of such investigations which can lead either to the loss of quality of the work, or to the examination of only a limited number of parameters of optimization.

Two-dimensional automatic method

In the case of deposits not very variable in one direction the intersection of the contours of the optimum pit C with a transverse vertical cross-section is identical to the result of the optimization carried out on this cross-section. The three-dimensional total pit is then obtained through interpolation between the cross-sections.

Cross-section optimization embodies the generation of a computer file containing the function $v(x,z)$ calculated at the intersections of a rectangular grid. The optimization algorithm is based upon simple notions of *dynamic programming* (analogous with the PERT method) and has been described by Lerchs and Grossmann (1965). It provides no problems of adaptation to electronic calculation irrespective of the size of the computer.

The *great simplicity of working* presents some drawbacks: lack of generality in use, restriction of the constraints of slopes to a system of grades homogeneous and isotropic, non-determination of the haulage road. However, this latter drawback is inherent in any method of automatic calculation. Indeed, the haulage road that fits best to the solution supplied by the computer is determined manually before the calculation of the discounted cashflow.

Three-dimensional automatic methods

A file $v(x,y,z)$ is assumed to contain the intersection points of a three-dimensional grid. Each point is the center of an elementary parallelepiped called for convenience 'cube'. Certain characteristics of the programs, such as the dimension of the manipulated files makes the use of *powerful computers* necessary. A computer at least like the IBM 360-50 and as far as possible like the IBM 360-91 or the CDC 6600 is needed.

The *algorithm of Lerchs and Grossmann* (1965) which is based upon the *theory of graphs* has been programmed. Its characteristics are as follows:

(i) The packing of the core memory of the computer is relatively important (eight bytes per point of the three-dimensional grid). Also, the time of calculation is important and cannot be estimated very accurately.

(ii) The configuration of the resultant open pit designs is roughly octagonal, because of the alternate simulation processes of constraints (the extraction of a cube must be preceded by the extraction of its five or nine neighbours in the immediately preceding step).

(iii) The selected system of borderline slopes must be homogeneous and isotropic, or be orthotropic (elliptical shape of the pits). As, in the actual cases, the rock mechanics investigation rarely indicates an isotropic borderline slope system, the undermentioned method is preferred.

A *geometrical algorithm* perfected and programmed by the Commissariat A L'energie Atomique is very easy to use. Contrary to the previous algorithm, it supplies the problem of optimization with an approximate solution, but the error in estimating v is insignificant. Indeed, so far no studied actual deposit has produced any significant error in optimization. The principles of the algorithm are described below.

The optimum open pit C consists of a *set of cones* of the same slope (the maximum allowable). The vertex of each cone is necessarily a payable cube. Any combination of payable cubes may be associated with the set of cones of which they are the vertices, that is, with an open pit technically workable: the optimum open pit is, when choosing among all the open pits made possible by all the possible combinations of payable cubes, that pit whose evaluation is the highest.

The cone whose vertex is a given payable cube is a set of payable and unpayable cubes. There is a simple rule to tell whether any given cube is part of the cone. If the product of its horizontal distance from the vertex and the admissible slope in the azimuth is less than its vertical distance from the vertex, then the cube is part of the cone. Thus *totally anisotropic* slopes may be analyzed without difficulty.

As the number of possible combinations of payable cubes is extremely high, a practical procedure has had to be perfected. A *fundamental property* has been used. As said previously, the optimum open pit C must be included in field D. If the search is limited to a subfield D', included completely in the field D, then the obtained open pit C' is included completely in the open pit C (Fig. 3). Thus, we can deduce a procedure for the optimization algorithm. *Investigate subfields*, each enclosed by the previous subfield and extract the corresponding sub-open pits by analyzing the valuations of the cubes which are included. The application of this procedure takes place at several levels:

(i) At any instant the studied subfield is limited at the base by a horizontal plane. The enlarging of this field downwards takes place only when the optimum open pit which is relative to it has been determined and extracted.

(ii) Inside a subfield we seek to initiate the successive extraction of sets of cones which can no longer include subsets of cones of positive evaluation. Any extraction of sets of cones initiates the re-examination of the previous subfields since the evaluations of some unpayable cubes may have been annulled.

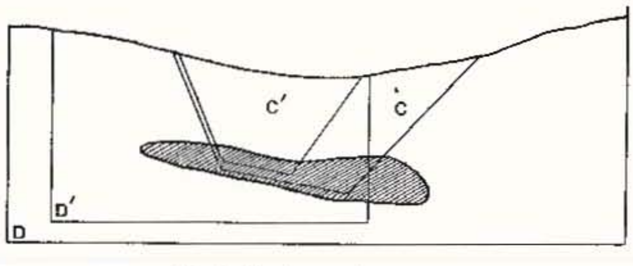

Fig. 3. Fundamental property:
If D' is included in D, C' is included in C.

Experience has shown that the examination of all the possible sets of a subfield leads to prohibitively long calculation times. Consequently, a compromise has been found between the rigorous optimization and an acceptable cost.

The advantages of this method seemed decisive to us: being able to evaluate *totally anistropic system of slopes* little filling of the core memory of the computer (four bytes per grid intersection), reasonable computer expenses — one hour on an IBM 360-91 for a field of 100 000 cubes (drawing of the open pit on Benson paper included).

The illustration of the open pit presented in Fig. 4 corresponds to the extraction of several tens of millions of cubic meters (the drawing which was originally printed automatically has been simplified for the needs of publication). Figure 5 defines the system of slopes chosen in this work, that is, the horizontal projection of the elementary cone and perspective view.

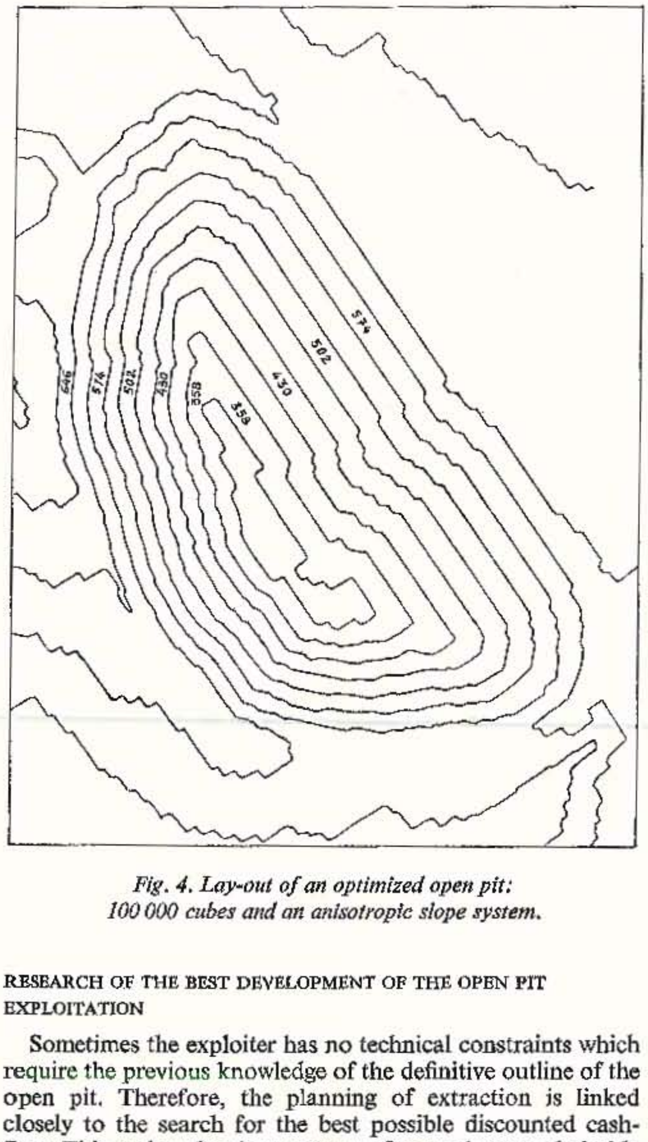

Fig. 4. Lay-out of an optimized open pit: 100 000 cubes and an anisotropic slope system.

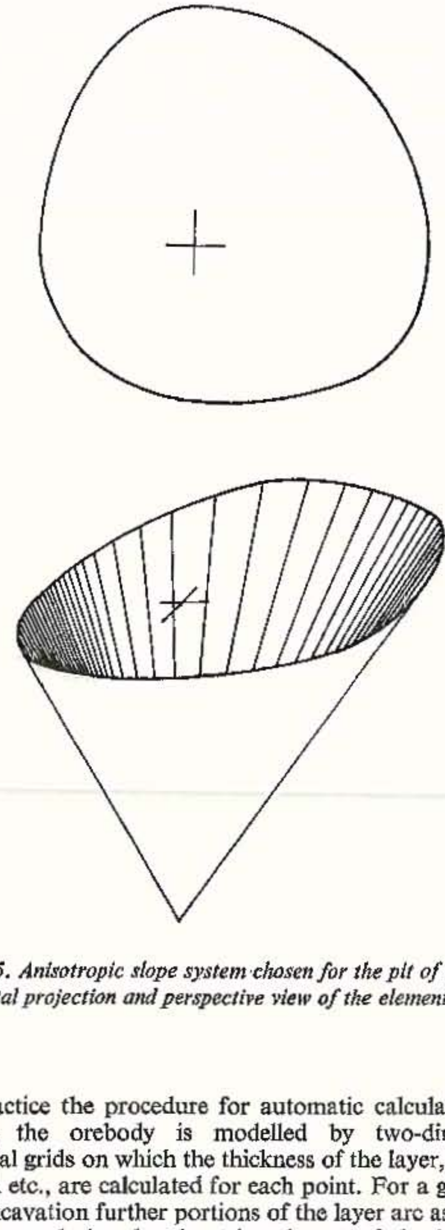

Fig. 5. Anisotropic slope system chosen for the pit of Fig. 4: horizontal projection and perspective view of the elementary cone.

RESEARCH OF THE BEST DEVELOPMENT OF THE OPEN PIT EXPLOITATION

Sometimes the exploiter has no technical constraints which require the previous knowledge of the definitive outline of the open pit. Therefore, the planning of extraction is linked closely to the search for the best possible discounted cash-flow. This makes the short-term profits much more desirable than the long-term profits. A project carried out by the Commissariat A L'energie Atomique on an open pit of several tens of millions of tons allowed the definition of such a plan through a simulation program of the working of the pit.

The deposit studied includes a small but thick layer dipping at 45 degrees. The exploiter defined a small open pit whose purpose was to assure the first months of production and which was situated in an area near the surface and was of high grade. To determine the working of the other parts of the deposit, simple rules have been chosen:

(i) the workings must be continuous, that is, extensions may be made by enlarging or deepening the existing excavation and

(ii) the increment chosen must result in a *stripping ratio* smaller than that produced by any other increment compatible with the previous constraint.

These rules applied not only to the choice of the first increment after extraction of the small open pit, but also from then on to the determination of all subsequent increments giving rise to the excavation existing at present.

In practice the procedure for automatic calculation is as follows: the orebody is modelled by two-dimensional horizontal grids on which the thickness of the layer, its grade, its depth etc., are calculated for each point. For a given state of the excavation further portions of the layer are assumed to be cut around the already stripped part of the layer. The algorithm of cone generation mentioned above permits the calculation of this stripping relative to each suggested portion of the layer and then the choice of the optimum method of achieving the increment. Human intervention is limited to a *check of the progression* in order to avoid impractical increments.

These calculations have been made particularly difficult by the existence of three different rocks with different properties (densities, slopes) in the overburden. Figure 6 shows the state of the open pit, as the simulation predicts it at different periods. Six-monthly, two-yearly and three-yearly states are shown.

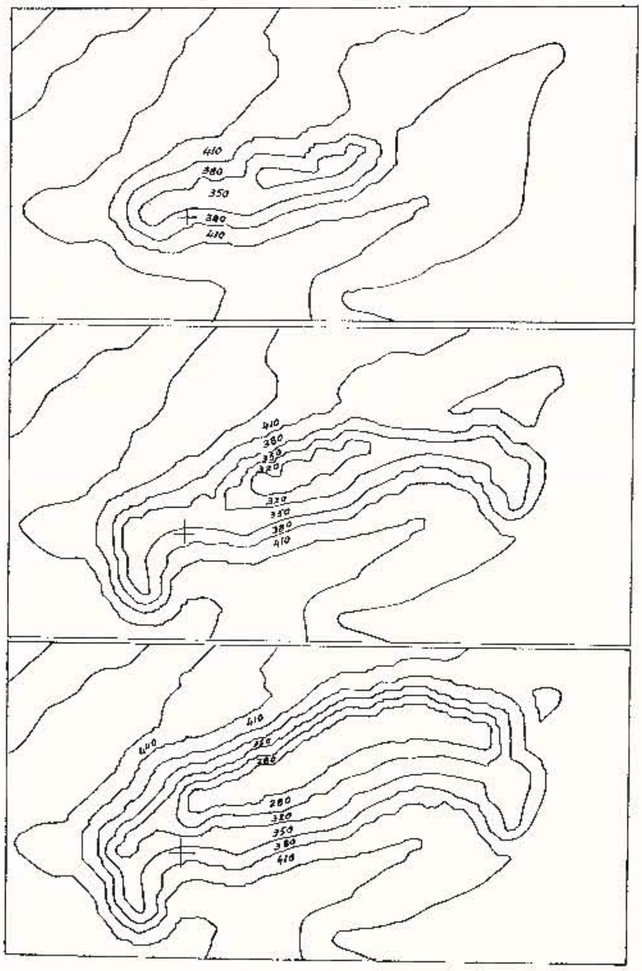

Fig. 6. Simulation of exploitation: state of the open pit at three different periods.

CONCLUSION

This paper demonstrates that a project of open pit exploitation must be carried out by using to the full all the available information, such as borehole logs, the geological model of the deposit, to optimize the open pit layout or to simulate exploitation and, finally, to complete economic evaluations by financial programs.

In these projects the computer possesses an *undeniable superiority over manual effort* since

(i) it avoids the small mistakes usually made in manual calculations,

(ii) it is relatively less costly for a given amount of work, and

(iii) it supplies the results much more rapidly; the influence of some important parameters, such as the metal prices, may be judged; probabilistic research may even lead to an estimate of the risk incurred by the exploiter.

However, the results obtained must at first be considered as indications which help the decision-makers to take their decisions.

APPENDIX A

AUTOMATIC CALCULATION OF GAMMA LOGGINGS

The gamma probing of a borehole, using a Geiger counter, gives an experimental function $F(z)$ of radium radiation, but this function is the sum of the radiation emitted by all radioactive elements of the rock.

If all radioactive elements are located in only one bed, it can be demonstrated that:

$$F = q\,A \qquad \ldots\ldots (1)$$

F = intensity of radioactivity measured by the Geiger counter in the borehole,

q = quantity of radioactive element in the bed, and

A = term dependent on geometrical data (borehole diameter, thickness of tubing, rock density, situation of the meter relative to the center of the bed).

If radioactive elements have any distribution in the rocks, (1) can be written:

$$F(z_0) = \int_{-\infty}^{+\infty} q(x)\,A(z_0 - x)\,dx \qquad \ldots\ldots (2)$$

$F(z_0)$ = intensity of radioactivity measured by the Geiger counter at depth z_0 in the borehole,

$q(x)$ = quantity of radioactive element at depth x in the borehole, and

$A(z_0 - z)$ = term depending on geometric data.

Equation (2) is a convolution product. $q(x)$ is found by deconvolution of $F(z_0)$.

Deconvolution is done by testing the distribution of $q(x)$ in N following slices of the same length (generally between 15 and 25 cm).

The main part of the deconvolution corresponds to an N-line, N-column matrix inversion. Calculation time (on IBM 360-91 computer) depends mainly on N:

$N = 50$ calculation time 2 sec.

$N = 100$ calculation time 10 sec.

$N = 220$ calculation time 100 sec.

To pass from radium to uranium grades, it is necessary to find only the connection between some uranium core analysis and the result of deconvolution of the corresponding gamma logs.

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